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Mathematics: understanding the score

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In September 2008, the Ofsted report *Mathematics: understanding the score* produced detailed evidence and analysis from inspections of mathematics teaching. The past decade has seen significant rises in standards in mathematics for pupils of all ages, although the rate of improvement has slowed at Key Stage 2 and stalled at Key Stage 1. It is clear from the report that children who start school with low levels of mathematical skills need to make good progress to close the gap and move forward confidently. Meanwhile, high attaining pupils require greater challenge in lessons: many primary teachers need stronger subject knowledge to do this well. It is of vital importance for pupils of all abilities to shift teaching and learning in mathematics away from a narrow emphasis on disparate skills towards a focus on pupils' mathematical understanding.

The fundamental issue for teachers is how better to develop that understanding. The essential ingredients of effective mathematics teaching are subject knowledge and understanding of the ways in which pupils learn mathematics – drawn together in the report as 'subject expertise' – together with experience of using these in the classroom. The report describes how the best teaching in both phases is enthusiastic, knowledgeable and focused clearly on developing pupils' understanding of important concepts. Good assessment throughout each lesson enables teachers to see how pupils are thinking and to adjust teaching and learning strategies accordingly. By developing pupils' mathematical independence, teachers also equip them for success in national tests and beyond.

The report highlighted many examples of good mathematics teaching and weaker practice. With this in mind Ofsted has produced this booklet to help teachers in providing the best possible opportunities for all pupils.

What are the essentials of good mathematics teaching?

The following list does not define what constitutes good or satisfactory teaching, but shows the difference between good and satisfactory features. Teaching that encompasses most of the good features may well be outstanding. Similarly, the cumulative effect of many weaker features can slow pupils' progress.

Features of good mathematics teaching	Features of satisfactory mathematics teaching
Lesson objectives involve understanding and make what is to be learned in the lesson very clear.	Lesson objectives are procedural, such as descriptions of work to be completed, or are general, such as broad topic areas.
Teaching successfully focuses on each pupil's learning. Pupils are clear about what they are expected to learn in the lesson and how to show evidence of this.	Teaching successfully focuses on teaching the content of the lesson. Pupils complete correct work and are aware of the lesson objectives but may not understand what they mean or what they need to do to meet them.
The lesson forms a clear part of a developmental sequence and pupils recognise links with earlier work, different parts of mathematics or contexts for its use.	The lesson stands alone adequately but links are superficial; for example, links are made with the previous lesson but not in a way that all the pupils understand.
Teachers introduce new terms and symbols meaningfully and expect and encourage correct use.	Teachers introduce new terms and symbols accurately and demonstrate correct spelling.
Whole class teaching/questioning: Pupils spend enough time listening to teachers' explanation and working to develop their understanding, and teachers move them on when appropriate. Teachers and support staff ensure all pupils participate actively in whole-class activity, such as through using mini whiteboards or partner discussions. When offering answers or accounts, the teacher expects pupils to give explanations of their reasoning as well as their methods. Pupils are challenged if their explanations do not reflect their ability.	**Whole class teaching/questioning:** Teachers give effective exposition but pupils' understanding is limited due to time constraints or not extended due to limitations with the task. Questioning and whole-class activities are pitched appropriately but do not involve all pupils actively; for example, few hands up, questions directed to few pupils, mini whiteboards held up whenever pupils are ready so not all give answers or some copy from others. Questioning is clear and accurate but does not require explanation or reasoning; pupils describe the steps in their method accurately but do not explain **why** it works.

Features of good mathematics teaching	Features of satisfactory mathematics teaching
Group/individual work: Teachers monitor all pupils' understanding throughout the lesson. They recognise quickly when pupils already understand the work or what their misconception might be. They extend thinking through building on pupils' contributions, questions and misconceptions to aid learning, flexibly adapting to meet needs and confidently departing from plans. The work challenges all pupils as it is informed by teachers' knowledge of pupils' learning; for example, through encouraging pupils capable of doing so to improve their explanations or use more efficient methods. Work requires thinking and reasoning and enables pupils to fully understand objectives. Pupils can explain why a method works and solve again a problem they have solved a few weeks earlier. Non-routine problems, open-ended tasks and investigations are used often by **all** pupils to develop the broader mathematical skills of problem solving, reasoning and generalising.	**Group/individual work:** Competent questioning but the teacher may miss opportunities to respond to needs; for example, does not build on errors or sticks too closely to plans. Pupils generally complete work correctly but may have made errors or already understand the work so tasks do not fully stretch the high attainers or support the low attainers. Methods are clearly conveyed by teachers and used accurately by pupils; pupils focus on obtaining correct answers rather than enhancing understanding and questions may not be carefully selected. Skills may be short-lived so pupils cannot answer questions which they have completed correctly a few weeks earlier. Typical lessons consist of routine exercises that develop skills and techniques adequately but pupils have few opportunities to develop reasoning, problem solving and investigatory skills, or only the higher attainers are given such opportunities.
Pupils develop independence and confidence by recognising when their solutions are correct and persevering to overcome difficulties because they expect to be able to solve problems; the teacher's interventions support them in estimating and checking for themselves.	Support generally offered to pupils does not develop independence in solving complete problems; for example, answers are given too readily or the problem is broken down so much that pupils do not know **why** the sequence of steps was chosen. Pupils may ask for help at each step and are given directed steps to take rather than interventions that encourage thinking and confidence that they can succeed.
Teaching assistants know the pupils well, are well briefed on the concepts and expected misconceptions, and provide support throughout the lesson that enhances thinking and independence.	Teaching assistants facilitate access of all pupils, though they may be less active in whole-class work.

Features of good mathematics teaching	Features of satisfactory mathematics teaching
Teachers (and pupils) have a good grasp of what has been learnt judged against criteria that they understand; this is shown through pupil discussion, reflection, oral or written summaries, and ascertained by the teacher's monitoring throughout the lesson.	Teachers (and pupils) make some accurate assessment of learning; for example, the teacher correctly reflects in a plenary what many pupils have achieved, pupils make an impressionistic assessment of their learning, such as using traffic lights or against a generic lesson objective.
Teachers' marking identifies errors and underlying misconceptions and helps pupils to overcome difficulties: for example, by setting clear targets, which pupils take responsibility for following up and seek to understand where they have gone wrong.	Accurate marking by the teacher identifies errors and provides pupils with feedback; important work has been marked by pupils or teacher.
Good use of subject knowledge to capitalise on opportunities to extend understanding, such as through links to other subjects, more complex situations or previously learned mathematics.	Any small slips or vagueness in use of subject knowledge do not prevent pupils from making progress.
Pupils exude enjoyment and involvement in the lesson. Pupils are confident enough to offer right and wrong comments. Pupils naturally listen to and respond to each other's comments, showing engagement with them.	Pupils enjoy making progress in an ordered environment. Some pupils offer responses to whole-class questions. Pupils listen to the teacher's and pupils' contributions and respond to them when asked to.

In practice

The following examples give some illustration of the prime and weaker aspects of teaching of mathematics in primary schools.

Good curricular planning provides pupils with opportunities to apply mathematics to a variety of interesting tasks, enabling them to choose approaches, reason and refine their thinking in the light of their solutions. Teachers encourage pupils to discuss mathematical problems in depth and this helps to build their confidence. In a primary school where developing pupils' understanding was promoted effectively, pupils were confident in 'thinking aloud' and were not afraid to have their mistakes used to help others.

Prime practice: discussion

An interesting approach to ratio and proportion with Year 6 pupils with lots of discussion.

The teacher engaged pupils throughout the lesson by incorporating many activities and encouraging discussion and argument in pairs until an answer was agreed. A reverse approach to solving problems was effective in getting pupils to think about clarity of expression. The teacher put one cup of fruit juice and two cups of water in a jug and one cup of fruit juice and three cups of water into another jug. The contents of both jugs were poured into a bowl, which, by then, contained 2,800ml of the mixture.

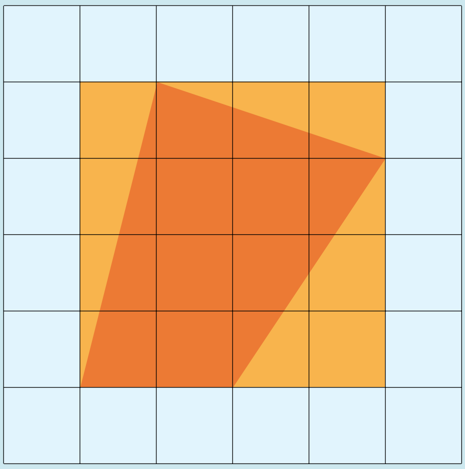
The teacher posed the question: how many millilitres of fruit juice are in the bowl? Pupils worked in pairs with jottings on mini whiteboards. Many struggled at first, argued with each other, but eventually worked out that $2/7$ of the mixture would be juice. Pupils were then asked to write a question, in words not just numbers, to match the problem they had just solved. As the lesson went on, middle-attaining pupils in the group completed more, similar questions and higher-attaining pupils were given some requiring much deeper thinking.

Conceptual approaches and practical activities promote understanding, allowing common misconceptions to surface and be tackled constructively. In the best lessons, teachers gave pupils opportunities to think for themselves, explain their reasoning and apply what they knew creatively. Listening to, observing and marking pupils' responses to rich prompts provided the teachers with useful evidence from which they diagnosed difficulties and the need for further challenge.

Prime practice: building understanding

Conceptual approaches to the teaching of area meant that Year 5 pupils could do much more than find the area of a rectangle using a formula.

A primary teacher emphasised that the area of a shape was measured by the number of 1cm squares it could hold. By drawing rectangles to the correct size on squared paper, she had helped pupils to give meaning to the numerical answers. They had initially counted squares. She checked carefully that pupils had recognised the rows and columns of squares in their rectangles and could use them to calculate the area of a rectangle more quickly. She introduced triangles and many other shapes through geo-boards. Pupils devised their own strategies for composite shapes, including halving to get triangles, and discussed them with other pupils.



By practising only one method at a time, pupils do not gain the confidence and intellectual flexibility they need. This can fragment the subject, because it is presented as a collection of apparently arbitrary rules for memorising. The rules can be incomplete or confusing.

Weaker factors: unhelpful rules

Teachers usually introduce rules to help pupils remember results or steps in methods. However, few are always true and many are never fully developed so that pupils understand the context of a rule. Here are two examples.

(a) 'To multiply by 10 you add a nought' but $3.4 \times 10 \neq 3.40$

Discussion about place value is the most powerful way of tackling multiplying by 10.

(b) 'Always measure from the end of the ruler' but this doesn't always work and is a common assumption young pupils make when learning to measure. Another error is that they measure from 1 on the scale.

The emphasis should be placed on measuring from 0, which is often at the end of a tape measure, but the scale on most rulers starts a little way in from the end of the ruler.

How might it be improved?

Where it is considered that rules might be useful, they should be unambiguous and developed with the pupils. The unthinking use of rules should be discouraged.

A key area for improvement is to develop more open-ended tasks that provide opportunities for pupils to investigate mathematically, for example, in choosing how to solve a complex task, or in exploring and making general statements.

Weaker factors: pseudo investigation

The way tasks are framed can close down opportunities for pupils to investigate mathematics. In this example, Year 5/6 pupils were nominally 'investigating' what happens when different combinations of odd and even numbers are subtracted. They had previously found rules for adding.

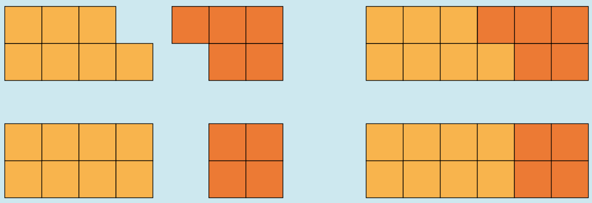
The teacher had presented the task as one of identifying 'the correct rule' by asking, 'Does odd minus odd give an odd or even answer?' Confident that a rule existed, pupils simply tried one example and inferred general rules from single examples.

The teacher's approach meant that pupils never engaged with the possibility that there might be no consistent rule. In the previous lesson they had been guided to record three rules for addition (O+O=E, O+E=O, E+E=E), but reasons why the rules worked and links between the rules were not made clear.

How might it be improved?

The teacher's questions could have been phrased in an open way, 'What happens when you add or subtract two odd numbers?' followed later by 'Does this always happen?'

Learning would have been better if the teacher had given the pupils greater independence by not assuming that a rule had to exist and by providing practical equipment such as interlocking cubes so that they could represent odd and even numbers visually. Pupils could then illustrate their explanations and justify rules. They could also have been encouraged to look for unifying ideas, for example, when adding two even or two odd numbers, the sum is always even.



The teacher might have benefited from guidance on teaching approaches for such tasks and about what aspects of using and applying mathematics pupils could develop through the activity.

Effective teachers anticipate pupils' likely misconceptions and are skilled in choosing resources and particular examples to expose misconceptions and check that their understanding is secure.

Prime practice: visual aids

Good use of a 10x10 grid with low-attaining Year 6 pupils helped understanding of tenths and hundredths and their fraction and decimal representations.

The teacher made excellent use of 10x10 grids on an interactive whiteboard to identify fractions 1/10 and 1/100, seamlessly moving to 0.1 and 0.01 and their equivalences with percentages. Pupils enjoyed using blank grids and the interactive whiteboard, for example when converting ½ to a percentage. They were excited by their success. The teacher also used the whiteboard exceptionally well to dispel misconceptions, for example when pupils suggested 8/10 is 8%.


Self Smart

One negative effect of interactive whiteboards is a reduction in pupils' use of practical equipment: generally, teachers do not use practical resources and games often enough to develop pupils' understanding of mathematical ideas and help them to make connections between different topics.

Weaker factors: visualisation

A Year 1 lesson about the properties of three-dimensional shapes was based on images displayed on the interactive whiteboard but gave no practical hands-on experience of the solids.

A teacher used an interactive whiteboard to teach Year 1 pupils about three-dimensional shapes. The pictures of the shapes caused confusion, between spheres and circles for example. Although pupils enjoyed a matching activity using the interactive whiteboard, they did not develop knowledge and understanding of the properties of three-dimensional shapes, such as the nature of the surfaces of a cone. The teacher did not adapt the teaching to take account of pupils' responses that showed their difficulties in using the two-dimensional representation of the three-dimensional shape.

How might it be improved?

Pupils would benefit from handling a range of real shapes so that they could feel and see the difference between two-dimensional and three-dimensional shapes. They could be encouraged to use their knowledge of properties of two-dimensional shapes to help describe the three-dimensional ones.

In the best examples, of provision in the Foundation Stage, the environment was mathematically rich with opportunities to explore ideas and practise skills. Many activities were practical, so learning was active and fun. An important element was the quality of adults' dialogue with the children; this was instrumental in their development. Young children often enjoyed trying to solve puzzles and problems, but only the better teaching required them to calculate or use the language of position rather than simply counting or naming shapes.

Prime practice: language development

Learning mathematical language in a Reception class. The children's language and conceptual understanding were developed securely through a range of well planned activities that provided plenty of opportunity for them to use new words, make comparisons and reason.

The teacher was working outside with a group of five children. They were wearing hard hats and were 'working' on a construction site, designing and building a house for the Three Billy Goats Gruff. They had a superb range of equipment from which to choose, including planks of various lengths and wooden blocks of different shapes and sizes. The teacher participated in their play, asking well phrased questions to develop and assess their understanding of shape, weight and length, such as 'Can you find a shorter plank than that one?' and 'Is it heavier than the other one or lighter?' She recorded the children's responses on a prepared sheet.

The activity was followed up well, using the interactive whiteboard and a program that showed pictures of different sized houses with three creatures of varying sizes alongside. The children were asked whether they thought the house would be better for the caterpillar, the dog or the giraffe, and were asked to explain why. All could offer good reasons: 'The giraffe's too tall, he wouldn't fit in.' 'It's a middle-sized house and the dog's middle-sized.' The children thoroughly enjoyed the activities, which also developed their gross motor skills, language, creative and social skills.

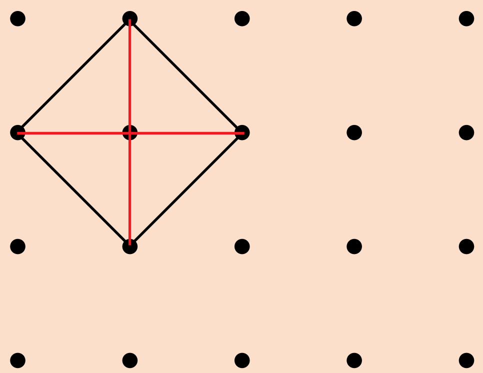
Other child-initiated activities included role play in the toy shop where children were pricing items and buying and selling them. They had the idea of using coins and giving change even though they did not fully understand the mathematics: 'Here's 1p'; 'Thank you – you need 1p back.'

The vast majority of primary teachers have little knowledge or experience of mathematics beyond courses they studied at school, such as GCSE or O-level mathematics. While they become familiar with the mathematics they are teaching, often to the same age group in consecutive years, the bigger picture of progression within the subject and the interrelationships between different aspects or topics can be lost, or sometimes never properly understood.

Weaker factors: knowledge of geometry

Primary teachers justifying whether a shape is a square or not.

As part of a professional development activity, groups of primary teachers drew various squares on dotted grids. In justifying whether the shape illustrated below was definitely a square, many offered approaches such as fitting corners of sheets of paper into the corners of the shape but this does not justify the angles being exactly 90°. When prompted, they could explain why the four sides were of equal length. Further coaxing led to identification of pairs of angles of 45°, and hence right angles.

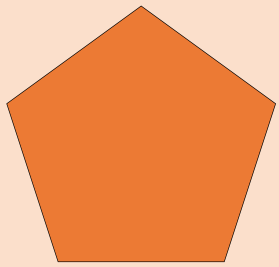


The teachers did not recognise the succinct proof gained by spotting that the diagonals were of equal length and bisected each other at right angles. Many did not recall this property of squares. Yet high-attaining pupils in Key Stage 2 are expected to use the properties of quadrilaterals to classify four-sided shapes. The mathematical knowledge and understanding of many teachers do not equip them to do this effectively.

Teachers sometimes make mistakes in their explanations or when demonstrating solutions. Such moments can trigger fruitful discussion and debate. However, when errors reflect teachers' weak understanding of mathematics and are not noticed or corrected, pupils can be left confused and in danger of repeating the error themselves. The security of their subsequent learning is also threatened. Limited subject knowledge restricts the dialogue teachers can have with pupils and the range of questions they can ask to probe pupils' understanding. As a result, teachers are less able to identify misconceptions and move pupils' learning on.

Weaker factors: gaps in subject knowledge

A Year 6 lesson on interior angles of polygons in which a teacher's weak subject knowledge led to pupils' incorrect understanding.



A Year 6 class was investigating the interior angles of regular polygons. Many found this difficult, but higher-attaining pupils had found that a pentagon has interior angles of 108°. The teacher said that this was not correct and encouraged them to divide 360° by 5 to get the answer, stating, 'the angles in any polygon add up to 360°'. This gave the answer of 72°, which puzzled the most able pupils as the interior angles were clearly bigger than right angles. Other pupils appeared to just accept the rule which they then incorrectly applied to other polygons.

How might it be improved?

The teacher had not realised that this was a gap in her knowledge. Possibly, she had confused previous knowledge about external angles which do sum to 360°. If she had had the confidence to ask the able pupils to explain their answer, she might have recognised her error. She returned to the pupils' answer of 108° in the next day's lesson.

Cross-curricular and other focus or themed days can add enjoyment and value to many pupils' learning in mathematics. For example, one primary school had a 'maths and art week' and another held a team problem-solving day.

Prime practice: real enrichment

Year 6 pupils investigate projects and bid for money from governors in the style of a popular television programme.

Groups of Year 6 pupils thought up ideas, consulted the rest of the school, and then planned their projects, including a healthy eating tuck shop and outdoor play. They carried out research through questionnaires, collated their findings, and used ICT very well. They researched costings, knowing they were expected to prove best value by comparing prices. The pupils devised criteria to ascertain which projects went forward to the judging panel, which comprised five governors, the chair of the Friends of the School and the headteacher. For this, they created presentations that gave a rationale, statistical analysis and justification for their project, including graphs and charts for visual impact, to convince the panel to part with their money.

All the groups were granted at least some of their funding and soon several schemes were in train. The pupils overcame practical problems as they arose, for example, acquiring old supermarket trolleys to customise into a tuck shop, helped in this design and technology project by a local secondary school. The project met its aims including the application of skills in calculation, problem-solving, communication, collaboration and ICT in a real-life context. Pupils enjoyed the contribution they made to the projects.

Generally, the frequency and quality of homework varied widely. The tasks set for homework concentrated on pupils practising taught skills. While this is important, since pupils need to be fluent in skills if they are to have the intellectual space for thinking when they tackle more complex or unusual problems, it should not be pupils' only experience of independent work.

Prime practice: primaries involving parents

Two examples of primary schools seeking to involve parents in supporting their child's learning in mathematics: a games library and a website.

A school was concerned that pupils' learning and confidence in applying numerical skills were not being reinforced sufficiently, especially at home. Formal homework was not seen to be the answer because the children were young. The teachers had the idea of a 'games library', accessible to all parents each week. Games were exchanged in the same way as library books. The games were colour-coded for age and ability so parents knew which were suitable for their children. The introduction of the library was considered to be a success story. Parents acted as librarians and the library was open on fixed days at the end of the afternoon, which was convenient for parents. The headteacher said that children loved having such a wide variety of games and that there had been a noticeable improvement in the progress and confidence of those who played with them with their parents. In addition, the children were becoming familiar with the idea of libraries as a valuable resource for themselves and adults.

Another school took guidance for parents and carers one step further than the usual information evenings. The pupils devised guidance on calculation strategies, such as doubling and halving. This was uploaded onto the school's website where it was supplemented by information on methods and progression in calculation.

Some of the report's examples of secondary mathematics lessons are pertinent to primary mathematics teaching. In the following example, the teacher did not assess the extent of pupils' difficulty accurately. The lesson may have been more effective if the teacher had circulated to check on pupils' books and discussions, posing questions to verify whether particular pupils who were likely to have difficulty did, in fact, understand.

The usual structure to primary lessons means that teachers do not often move around the class during the main part of the lesson when pupils are working in groups. This is because a couple of the groups are the planned focus of attention of the teacher and the teaching assistant, while the other pupils are expected to work independently. However, at this stage of the lesson their progress is in danger of stalling or slowing, either because they get stuck or because the work is undemanding.

Weaker factors: teacher not circulating

A lesson starter in which the teacher was unaware that pupils' progress was very variable.

A low-attaining Year 7 class was given a worksheet as a quick lesson starter. It contained several questions of the form 400 + 300 = 600 +…

The teacher did not circulate to check anyone's work so did not realise that some pupils had written 1,300 and attempted the remaining questions incorrectly as additions. While some pupils finished very quickly, others had managed only a few questions. The speed of responses showed that the pupils who already knew how to do this work were not extended and those who did not know gained little benefit.

How might it be improved?

If the teacher had moved around the class quickly checking pupils' first answers, or used mini whiteboards for the starter activity, he would have identified those pupils who were making the mistake of adding the three numbers. Continuing to circulate as pupils worked would show the teacher who was struggling and who was not challenged by the task.

Learning may have been better if the questions had been tailored to pupils' prior attainment, perhaps through two or three worksheets at different levels of challenge.

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